

# THE AMPLITUDE OF LEPTON-PAIR PRODUCTION IN RELATIVISTIC ION COLLISIONS BEYOND THE PERTURBATIVE THEORY

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## ABSTRACT

We investigate the structure of some first terms of Watson series representing the amplitude of lepton-pair production in the ion collisions. It is shown that infrared instabilities of individual terms trend to cancel each other providing the infrared stability of whole amplitude.

## 1 Introduction

The structure of the amplitude of the reaction

$$Z_1 + Z_2 \rightarrow Z_1 + Z_2 + e^+ + e^- \quad (1)$$

beyond the Born approximation was widely discussed in literature in past few years [1-12].

It is generally believed that it becomes extremely simple in ultrarelativistic limit due to practically full contraction of electromagnetic fields of colliding ions.

The idea to use solution of Dirac equation in the superposition of these fields for deriving of amplitude of reaction (1) was dominant in literature for a long time. The extra simple solution of problem under the consideration obtained in this way have been reported in several papers [1-3]. Later it have been proved [5,12] that this solution is incorrect. Detailed discussion of this issue can be found in [7].

The another approach to this problem [8,9,11,12] is based on the exploiting of more familiar technique of Feynman diagrams (*FD*) for the systematic perturbation calculations of the corrections to the Born approximation result. The authors [8,9] have proceeded in the summation of the contributions of the simplest infinite subsets of FD to  $e^+e^-$ -production amplitude, but met serious difficulties while attempting to generalize these results to the more complicated cases.

Partly this problem have been solved in the recent paper [13], where the Watson type representation of amplitude of reaction (1) in terms of amplitudes of  $e^\pm - Z_{1(2)}$ -scattering was derived. The further simplification of this amplitude achieved when one takes into account the requirement of its infrared stability can be.

## 2 Watson series and the problem of infrared stability

The amplitude of reaction (1) reads

$$A = \bar{u}(p_2) T(p_2, -p_1) v(p_1) \quad (2)$$

where  $p_1$  and  $p_2$  are the four momenta of positron and electron respectively;  $v(p_1)$  and  $u(p_2)$  are their bispinors and the matrix  $T(p_2, -p_1)$  is represented by Watson series

$$\begin{aligned} T(p_2, -p_1) = & T_1(p_2, -p_1) + T_2(p_2, -p_1) \\ & - \int d^4k T_1(p_2, k) G(k) T_2(k, -p_1) - \int d^4k T_2(p_2, k) G(k) T_1(k, -p_1) \\ & + \int d^4k_1 d^4k_2 T_1(p_2, k_1) G(k_1) T_2(k_1, k_2) G(k_2) T_1(k_2, -p_1) \\ & + \int d^4k_1 d^4k_2 T_2(p_2, k_1) G(k_1) T_1(k_1, k_2) G(k_2) T_2(k_2, -p_1) + \dots \end{aligned} \quad (3)$$

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or in the short notation

$$T = T_1 + T_2 - T_1 \otimes G \otimes T_2 - T_2 \otimes G \otimes T_1 \\ + T_1 \otimes G \otimes T_2 \otimes G \otimes T_1 + T_2 \otimes G \otimes T_1 \otimes G \otimes T_2 + \dots \quad (4)$$

above

$$G(k) = \frac{1}{(2\pi)^4} \frac{\hat{k}+m}{k^2-m^2+i0}, \quad (5)$$

$$T_1(p, p') = (2\pi)^2 \delta(p_+ - p'_+) \left[ \theta(p_+) f_1^{(+)}(\vec{p}_T - \vec{p}'_T) - \theta(-p_+) f_1^{(-)}(\vec{p}_T - \vec{p}'_T) \right] \gamma_+, \\ T_2(p, p') = (2\pi)^2 \delta(p_- - p'_-) \left[ \theta(p_-) f_2^{(+)}(\vec{p}_T - \vec{p}'_T) - \theta(-p_-) f_2^{(-)}(\vec{p}_T - \vec{p}'_T) \right] \gamma_-, \\ p_{\pm} = p_0 \pm p_z, \quad (6)$$

$$\gamma_{\pm} = \gamma_0 \pm \gamma_z,$$

$$\hat{k} = k_{\mu} \gamma_{\mu};$$

$$f_1^{(\pm)}(\vec{q}) = \frac{i}{2\pi} \int d^2b e^{i\vec{q} \cdot \vec{b}} \left[ 1 - S_1^{(\pm)}(\vec{b}, \vec{B}_1) \right], \\ f_2^{(\pm)}(\vec{q}) = \frac{i}{2\pi} \int d^2b e^{i\vec{q} \cdot \vec{b}} \left[ 1 - S_2^{(\pm)}(\vec{b}, \vec{B}_2) \right], \quad (7)$$

$$\vec{q} = \vec{p}_T - \vec{p}'_T, \\ S_1^{(\pm)}(\vec{b}, \vec{B}_1) = e^{\pm i\chi_1(\vec{b}, \vec{B}_1)}, \\ S_2^{(\pm)}(\vec{b}, \vec{B}_2) = e^{\pm i\chi_2(\vec{b}, \vec{B}_2)}, \quad (8)$$

$$\chi_1(\vec{b}, \vec{B}_1) = e \int_{-\infty}^{\infty} \Phi_1 \left( \sqrt{(\vec{b} - \vec{B}_1)^2 + z^2} \right) dz, \\ \chi_2(\vec{b}, \vec{B}_2) = e \int_{-\infty}^{\infty} \Phi_2 \left( \sqrt{(\vec{b} - \vec{B}_2)^2 + z^2} \right) dz. \quad (9)$$

Here  $\Phi_{1,2}(r)$  are the Coulomb potentials of ions  $Z_1$  and  $Z_2$  respectively,  $\vec{B}_1$  and  $\vec{B}_2$  are their impact parameters.

Since integrals defining the phase shifts  $\chi_{1,2}$  are divergent in the unscreened case, the infinitesimal screening parameters  $\lambda_{1,2}$  are usually introduced as

$$\Phi_1(r) = \lim_{\lambda_1 \rightarrow 0} \frac{Z_1 e \exp(-\lambda_1 r)}{r} \\ \Phi_2(r) = \lim_{\lambda_2 \rightarrow 0} \frac{Z_2 e \exp(-\lambda_2 r)}{r} \quad (10)$$

to make  $\chi_{1,2}$  finite.

If some calculated quantity, depending on  $\lambda_{1,2}$ , approach the definite limit when  $\lambda_{1,2}$  are turned to zero, then this quantity is called infrared stable. In opposite case it is known as infrared unstable.

Introduced above S -matrices  $S_{1(2)}$  are infrared unstable themselves, but their products

$$S_1^{(+)}(\vec{b}, \vec{B}_1) S_1^{(-)}(\vec{b}', \vec{B}_1)$$

and

$$S_2^{(+)}(\vec{b}, \vec{B}_2) S_2^{(-)}(\vec{b}', \vec{B}_2)$$

are infrared stable.

Each term of Watson expansion contains infrared unstable products of  $S_{1,2}^{(\pm)}$ -matrices, but only some of these terms contain also the infrared stable contributions of them. Consequently, each term of Watson expansion is infrared unstable itself. But sum of all these terms, representing the amplitude (2), must be infrared stable.

This means that infrared unstable parts of different terms of Watson expansion must mutually cancel each other. To see this, we need make some calculations.

### 3 The simplest infrared stable $e^+e^-$ -production amplitude

Let us represent the amplitude  $A$  in the form

$$A = A_1 + A_2 - A_{12} - A_{21} + A_{121} + A_{212} - A_{1212} - A_{2121} + \dots \quad (11)$$

where, say,  $A_{121}$  corresponds to the term  $T_1 \otimes G \otimes T_2 \otimes G \otimes T_1$  in the Watson expansion (4).

The simple dependence of  $T_{1,2}(p, p')$  on light-cone components of momenta  $p, p'$  allows to make explicitly all integrations over light-cone components of intermediate momenta with following results:

$$\begin{aligned} A_{12} &= \frac{1}{2} \int d^2k a_{12}(\vec{k}) f_1^{(+)}(\vec{q}_1) f_2^{(-)}(\vec{q}_2), \\ A_{21} &= \frac{1}{2} \int d^2k a_{21}(\vec{k}) f_2^{(+)}(\vec{q}_1) f_1^{(-)}(\vec{q}_2), \end{aligned} \quad (12)$$

where

$$\begin{aligned} a_{12}(\vec{k}) &= \frac{\bar{u}(p_2) \gamma_+ \nu(\vec{k}) \gamma_- v(p_1)}{p_{2+} p_{1-} + \mu(\vec{k})}, \\ a_{21}(\vec{k}) &= \frac{\bar{u}(p_2) \gamma_- \nu(\vec{k}) \gamma_+ v(p_1)}{p_{2-} p_{1+} + \mu(\vec{k})}, \\ \nu(\vec{k}) &= m - \vec{\gamma}_T \vec{k}, \quad \mu(\vec{k}) = m^2 + \vec{k}^2, \\ \vec{q}_1 &= \vec{p}_{2T} - \vec{k}, \quad \vec{q}_2 = \vec{p}_{1T} + \vec{k}; \end{aligned} \quad (13)$$

$$\begin{aligned}
A_{121} &= \frac{1}{(4\pi)^2} \int d^2 k_1 d^2 k_2 \left\{ a_{121}^{(+)}(\vec{k}_1, \vec{k}_2) f_1^{(+)}(\vec{q}_1) f_2^{(+)}(\vec{q}_2) f_1^{(-)}(\vec{q}_3) \right. \\
&\quad \left. + a_{121}^{(-)}(\vec{k}_1, \vec{k}_2) f_1^{(+)}(\vec{q}_1) f_2^{(-)}(\vec{q}_2) f_1^{(-)}(\vec{q}_3) \right\}, \\
A_{212} &= \frac{1}{(4\pi)^2} \int d^2 k_1 d^2 k_2 \left\{ a_{212}^{(+)}(\vec{k}_1, \vec{k}_2) f_2^{(+)}(\vec{q}_1) f_1^{(+)}(\vec{q}_2) f_2^{(-)}(\vec{q}_3) \right. \\
&\quad \left. + a_{212}^{(-)}(\vec{k}_1, \vec{k}_2) f_2^{(+)}(\vec{q}_1) f_1^{(-)}(\vec{q}_2) f_2^{(-)}(\vec{q}_3) \right\},
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
a_{121}^{(\pm)}(\vec{k}_1, \vec{k}_2) &= -a_{121}(\vec{k}_1, \vec{k}_2) \left\{ \ln \left[ \frac{\mu(\vec{k}_1) p_{1+}}{\mu(\vec{k}_2) p_{2+}} \right] \pm \pi i \right\}, \\
a_{121}(\vec{k}_1, \vec{k}_2) &= \frac{\bar{u}(p_2) \gamma_+ \nu(\vec{k}_1) \gamma_- \nu(\vec{k}_2) \gamma_+ v(p_1)}{p_{1+} \mu(\vec{k}_1) + p_{2+} \mu(\vec{k}_2)}, \\
a_{212}^{(\pm)}(\vec{k}_1, \vec{k}_2) &= -a_{212}(\vec{k}_1, \vec{k}_2) \left\{ \ln \left[ \frac{\mu(\vec{k}_1) p_{1-}}{\mu(\vec{k}_2) p_{2-}} \right] \pm \pi i \right\}, \\
a_{212}(\vec{k}_1, \vec{k}_2) &= \frac{\bar{u}(p_2) \gamma_- \nu(\vec{k}_1) \gamma_+ \nu(\vec{k}_2) \gamma_- v(p_1)}{p_{1-} \mu(\vec{k}_1) + p_{2-} \mu(\vec{k}_2)},
\end{aligned} \tag{15}$$

$$\vec{q}_1 = \vec{p}_{2T} - \vec{k}_1, \quad \vec{q}_2 = \vec{k}_1 - \vec{k}_2, \quad \vec{q}_3 = \vec{p}_{1T} + \vec{k}_2;$$

$$A_{1212} = A_{1212}^{(s)} + A_{1212}^{(u)}, \tag{16}$$

where

$$\begin{aligned}
A_{1212}^{(s)} &= \int d^2 k_1 d^2 k_2 d^2 k_3 a_{1212}^{(s)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) f_1^{(+)}(\vec{q}_1) f_2^{(+)}(\vec{q}_2) f_1^{(-)}(\vec{q}_3) f_2^{(-)}(\vec{q}_4), \\
A_{1212}^{(u)} &= \int d^2 k_1 d^2 k_2 d^2 k_3 a_{1212}^{(u)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) f_1^{(+)}(\vec{q}_1) \left[ f_2^{(+)}(\vec{q}_2) + f_2^{(-)}(\vec{q}_2) \right] \\
&\quad \times \left[ f_1^{(+)}(\vec{q}_3) + f_1^{(-)}(\vec{q}_3) \right] f_2^{(-)}(\vec{q}_4);
\end{aligned} \tag{17}$$

$$A_{2121} = A_{2121}^{(s)} + A_{2121}^{(u)}, \tag{18}$$

where

$$\begin{aligned}
A_{2121}^{(s)} &= \int d^2 k_1 d^2 k_2 d^2 k_3 a_{2121}^{(s)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) f_2^{(+)}(\vec{q}_1) f_1^{(+)}(\vec{q}_2) f_2^{(-)}(\vec{q}_3) f_1^{(-)}(\vec{q}_4), \\
A_{2121}^{(u)} &= \int d^2 k_1 d^2 k_2 d^2 k_3 a_{2121}^{(u)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) f_2^{(+)}(\vec{q}_1) \left[ f_1^{(+)}(\vec{q}_2) + f_1^{(-)}(\vec{q}_2) \right] \\
&\quad \times \left[ f_2^{(+)}(\vec{q}_3) + f_2^{(-)}(\vec{q}_3) \right] f_1^{(-)}(\vec{q}_4),
\end{aligned} \tag{19}$$

$$\begin{aligned}
a_{1212}^{(s)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) &= a_{1212}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \left\{ -\ln \left[ \frac{\mu(\vec{k}_1) \mu(\vec{k}_3)}{\mu(\vec{k}_2) p_{2+} p_{1-}} \right] - i\pi \right\}, \\
a_{1212}^{(u)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) &= a_{1212}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \frac{1}{3} \left\{ \ln^2 \left[ \frac{\mu(\vec{k}_1) \mu(\vec{k}_3)}{\mu(\vec{k}_2) p_{2+} p_{1-}} \right] + \pi^2 \right\},
\end{aligned}$$

$$\begin{aligned}
a_{2121}^{(s)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) &= a_{2121}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \left\{ -\ln \left[ \frac{\mu(\vec{k}_1) \mu(\vec{k}_3)}{\mu(\vec{k}_2) p_{2-p_{1+}}} \right] - i\pi \right\}, \\
a_{2121}^{(u)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) &= a_{2121}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \frac{1}{3} \left\{ \ln^2 \left[ \frac{\mu(\vec{k}_1) \mu(\vec{k}_3)}{\mu(\vec{k}_2) p_{2-p_{1+}}} \right] + \pi^2 \right\}, \\
a_{1212}(\vec{k}_1, \vec{k}_2, \vec{k}_3) &= \frac{\bar{u}(p_2) \gamma_{+\nu}(\vec{k}_1) \gamma_{-\nu}(\vec{k}_2) \gamma_{+\nu}(\vec{k}_3) \gamma_{-v}(p_1)}{\mu(\vec{k}_1) \mu(\vec{k}_3) + \mu(\vec{k}_2) p_{2+p_{1-}}}, \\
a_{2121}(\vec{k}_1, \vec{k}_2, \vec{k}_3) &= \frac{\bar{u}(p_2) \gamma_{-\nu}(\vec{k}_1) \gamma_{+\nu}(\vec{k}_2) \gamma_{-\nu}(\vec{k}_3) \gamma_{+v}(p_1)}{\mu(\vec{k}_1) \mu(\vec{k}_3) + \mu(\vec{k}_2) p_{2-p_{1+}}}, \tag{20}
\end{aligned}$$

$$\vec{q}_1 = \vec{p}_{2T} - \vec{k}_1, \quad \vec{q}_2 = \vec{k}_1 - \vec{k}_2, \quad \vec{q}_3 = \vec{k}_2 - \vec{k}_3, \quad \vec{q}_4 = \vec{p}_{1T} + \vec{k}_3.$$

Substituting in these expressions  $f_{1,2}^{(\pm)}$ -amplitudes in the form

$$\begin{aligned}
f_1^{(\pm)}(\vec{q}) &= 2\pi i \left( \delta(\vec{q}) - S_1^{(\pm)}(\vec{q}, \vec{B}_1) \right), \\
f_2^{(\pm)}(\vec{q}) &= 2\pi i \left( \delta(\vec{q}) - S_2^{(\pm)}(\vec{q}, \vec{B}_2) \right), \tag{21}
\end{aligned}$$

where

$$\begin{aligned}
S_1^{(\pm)}(\vec{q}, \vec{B}_1) &= \frac{1}{(2\pi)^2} \int d^2b \exp(i\vec{q}\vec{b}) S_1^{(\pm)}(\vec{b}, \vec{B}_1), \\
S_2^{(\pm)}(\vec{q}, \vec{B}_2) &= \frac{1}{(2\pi)^2} \int d^2b \exp(i\vec{q}\vec{b}) S_2^{(\pm)}(\vec{b}, \vec{B}_2), \tag{22}
\end{aligned}$$

one obtains the amplitude (2) in terms of products  $S_{1(2)}^{(\pm)}$ -matrices.

Using the following identities

$$\begin{aligned}
\gamma_{\pm\nu}(-\vec{p}_{1T}) \gamma_{\mp\nu}(p_1) &\equiv -2p_{1\pm} \gamma_{\pm\nu}(p_1), \\
\bar{u}(p_2) \gamma_{\pm\nu}(\vec{p}_{2T}) \gamma_{\mp\nu} &\equiv 2p_{2\pm} \bar{u}(p_2) \gamma_{\pm\nu}, \\
\gamma_{\pm\nu}(\vec{k}) \gamma_{\mp\nu}(\vec{k}) \gamma_{\pm\nu} &\equiv 4\mu(\vec{k}) \gamma_{\pm\nu}, \tag{23}
\end{aligned}$$

one can easily check that the sum

$$\tilde{A} = A_{12} + A_{21} - A_{121} - A_{212} + A_{1212}^{(s)} + A_{2121}^{(s)} \tag{24}$$

is infrared stable and reads

$$\begin{aligned}
\tilde{A} &= \frac{i\pi}{4} \int d^2k_1 d^2k_2 d^2k_3 a_{1212}^{(s)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \\
&\times \left\{ \delta(\vec{q}_1) \delta(\vec{q}_2) \delta(\vec{q}_3) \delta(\vec{q}_4) - S_1^{(+)}(\vec{q}_1, \vec{B}_1) S_2^{(+)}(\vec{q}_2, \vec{B}_2) S_1^{(-)}(\vec{q}_3, \vec{B}_1) S_2^{(-)}(\vec{q}_4, \vec{B}_1) \right\} \\
&+ \frac{i\pi}{4} \int d^2k_1 d^2k_2 d^2k_3 a_{2121}^{(s)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \\
&\times \left\{ \delta(\vec{q}_1) \delta(\vec{q}_2) \delta(\vec{q}_3) \delta(\vec{q}_4) - S_2^{(+)}(\vec{q}_1, \vec{B}_2) S_1^{(+)}(\vec{q}_2, \vec{B}_1) S_2^{(-)}(\vec{q}_3, \vec{B}_2) S_1^{(-)}(\vec{q}_4, \vec{B}_1) \right\}. \tag{25}
\end{aligned}$$

Thus, all infrared unstable terms in (24) cancel each other.

The rest

$$A_{1212}^{(u)} + A_{2121}^{(u)}$$

is of order  $(Z_1\alpha \cdot Z_2\alpha)^3$ , as a  $A_{121212}$  and  $A_{212121}$ , which is beyond of this consideration, and we hope that its infrared unstable components must be canceled with similar unstable expressions of higher order terms, like was shown above for lower order case. We plan to consider this issue elsewhere.

## 4 Conclusion

The Watson-type representation for the amplitude of production of  $e^+e^-$ -pair in the ion-ion collisions can be considered as a resummation of FD of perturbation theory in more economic form. The application of the requirement of infrared stability of this amplitude provides new tool for future simplification.

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